Unpacked South Dakota State Mathematics Standards

Purpose: *In order for students to have the best chance of success, standards, assessment, curriculum resources, and instruction must be aligned in focus, coherence, and rigor. Unpacked standards documents are intended to help align instruction to the focus, coherence, and rigor of the South Dakota State Mathematics Standards. The standards have been organized in clusters as they are not so much built from topics, but rather woven out of progressions. Not all content in a given grade is emphasized equally in the mathematics standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting standards will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.*

Domain: Numbers and Operations - Fractions	**Grade Level: 4**

4.NF.A Cluster: Extend understanding of fraction equivalence and ordering.

This cluster asks learners to extend their work with equivalent fractions. They use models to find patterns in equivalent patterns and eventually generate a rule for finding equivalent fractions. They will extend their knowledge of comparing fractions from like numerators and denominators to comparing fractions with unlike numerators and denominators.

****This is a MAJOR cluster.*** *Students should spend the large majority of their time (65-85%) on the major work of the grade. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.*

4.NF.1 - Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4.NF.2 - Compare two fractions with different numerators and different denominators, by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols <, >, =, and justify the conclusions.

Aspects of Rigor: (Conceptual, Procedural, and/or Application)

Conceptual Understanding	***Procedural Fluency***	***Application***
Using models, explain how number and size of the fraction parts can differ but are still equal. **(4.NF.1)**	Recognize when fractions are equivalent when the number and size of the fraction parts are different. **(4.NF.1)** Create equivalent fractions when the number and size of the fraction parts are different. **(4.NF.1)**	
Understand how to convert two fractions to have common denominators. **(4.NF.2)** Understand how to convert two fractions to have common numerators. **(4.NF.2)**	Convert fractions to have common denominators. **(4.NF.2)** Convert fractions to have common numerators. **(4.NF.2)** Compare two fractions with different numerators and denominators. **(4.NF.2)** Use symbols (<, >, =) to compare two	

	fractions. **(4.NF.2)**	

Enacting the Mathematical Practices - Evidence of Students Engaging in the Practices

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
   - Construct mathematical arguments to explain their thinking as they build sets of equivalent fractions.
4. **Model with mathematics.**
   - Create visual models of equivalent fractions to build understanding
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
   - Look for fundamental structures of fractional numbers that will be critical for future fraction work.
8. **Look for and express regularity in repeated reasoning.**
   - Students will find and discuss patterns as they find equivalent fractions.

Vertical and Horizontal Coherence and Learning Progressions

Previous Learning Connections	***Current Learning Connections***	***Future Learning Connections***
In third grade denominators are limited to 2, 3, 4, 6, and 8 Represent whole numbers as equivalent fractions(3/3=1 and 4/1=4) **(3.NF.3)** Understand equivalent fractions as the same quantity with different names **(3.NF.3)** Compare two fractions with the same numerator or the same denominator by reasoning about their size **(3.NF.3d)** Understand that decomposing into more equal shares creates smaller shares **(1.G.3)**	Compare and order fractions by generating equivalent fractions **(4.NF.2)** Replace mixed numbers with equivalent fractions**(4.NF.3c)** Express a fraction with denominator 10 as an equivalent fraction with denominator 100 **(4.NF.5)** Compare two decimals to hundredths by reasoning about their sizes **(4.NF.7)**	Add and subtract with unlike denominators **(5.NF.1) (5.NF. 2)** Interpret a fraction as division of the numerator by the denominator. **(5.NF.3)** Relate the principle of fraction equivalence **(5.NF.5b)** Generate equivalent ratios and compare ratios **(6.RP.3)**

Vocabulary *(Key Terms Used by Teachers and Students in this Cluster):*

- Fraction
- Unit fraction
- Equivalent
- Multiple
- Denominator
- Numerator
- Comparison/compare
- ‹, ›, =
- Benchmark fraction
- Common denominators
- Convert

Relevance, Explanations, and Examples:

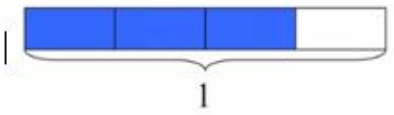
In 4th grade denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100.

4.NF.1

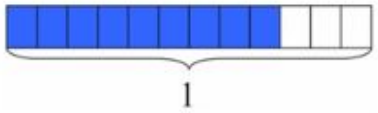
- Use a variety of visual area and linear fraction models to recognize and generate equivalent fractions.
- Explain connections between models and multiplying the numerator and denominator by the same number.

Task

a. The rectangle below has length 1. What fraction does the shaded part represent?



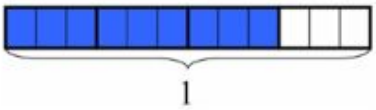
b. The rectangle below has the same length as the rectangle above. What fraction does the shaded part represent?



c. Use the pictures to explain why the two fractions represented above are equivalent.

Answer

c. Three pieces in the bottom rectangle have the same size as 1 piece in the top rectangle. We can even show this by darkening the lines around groups of three small pieces in the rectangle that represents $\frac{9}{12}$:



When we make groups of three in the bottom rectangle, there are 3 groups of 3 shaded pieces and 4 groups of 3 in the whole rectangle. Using these groups, we see that

$$\frac{9}{12} = \frac{(3 \times 3)}{(4 \times 3)} = \frac{3}{4}$$

of the bottom rectangle is shaded. Since the shaded portion is the same in each case but we just look at it in a different way and describe it with a different fraction, the fractions are equal. So

$$\frac{9}{12} = \frac{3}{4}$$

This standard addresses equivalent fractions by examining the idea that equivalent fractions can be created by multiplying both the numerator and denominator by the same number or by dividing a shaded region into various parts.
Example:

½ = 2/4 = 6/12

	1/2		
1/4	2/4		

1/12	4/12		
2/12	5/12		
3/12	6/12		

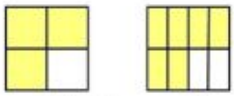
4.NF.2

- Create equivalent fractions that have common numerators, and then compare the denominators or the size of the fractional parts to compare the fractions (Example: Have students explore ways of comparing 2/3 and 4/5 by finding a common numerator.)
- Create equivalent fractions that have common denominators, and then compare the numerators or number of parts being considered (Example: have students explore ways of comparing 2/3 and 4/5 by finding a common denominator.)

Example: There are two cakes on the counter that are the same size. The first cake has ½ of it left. The second cake has 5/12 of it left. Which cake has more left?
Verbal Explanation: I know that 6/12 equal ½. Therefore, the second cake which has 5/12 left is less than ½.

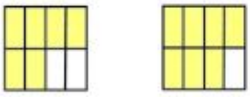
Comparing $\frac{3}{4}$ and $\frac{7}{8}$

Changing $\frac{3}{4}$



$\frac{3}{4} \overset{\times 2}{=} \frac{6}{8}$

Comparing $\frac{3}{4}$ and $\frac{7}{8}$

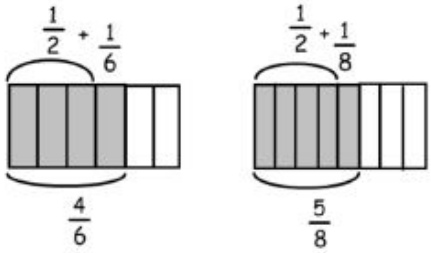


$\frac{3}{4} = \frac{6}{8} < \frac{7}{8}$

$\frac{7}{8} < \frac{13}{12}$

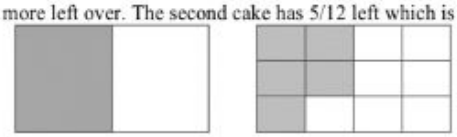
Students reason using benchmarks such as 1/2 and 1. For example, they see that 7/8 is less than 13/12 because 7/8 is less than 1 (and is therefore to the left of 1 on a number line), but 13/12 is greater than 1 (and is therefore to the right of 1 on a number line).

When using the benchmark fraction of $\frac{1}{2}$ to compare $\frac{4}{6}$ and $\frac{5}{8}$ you can use bar models such as these:



$\frac{4}{6}$ is $\frac{1}{6}$ larger than $\frac{1}{2}$, while $\frac{5}{8}$ is $\frac{1}{8}$ larger than $\frac{1}{2}$. Since $\frac{1}{6}$ is larger than $\frac{1}{8}$, $\frac{4}{6}$ is the greater fraction.

Area model:
The first cake has more left over. The second cake has 5/12 left which is smaller than ½.



Achievement Level Descriptors	
Cluster: **Extend understanding of fraction equivalence and ordering.**	
Concepts and Procedures	***Level 1:*** Students should be able to recognize that fraction comparisons are valid only when the two fractions are referring to the same whole.
	Level 2: Students should be able to compare two fractions with different numerators and different denominators using <, >, and = by comparing to a benchmark fraction such as 1/2 and recognize equivalent fractions using visual models.
	Level 3: Students should be able to extend understanding to compare two fractions with different numerators and different denominators using <, >, and = by creating common denominators or numerators and recognize and generate equivalent fractions using visual models.
	Level 4: Students should be able to extend understanding to compare two fractions with different numerators and different denominators using <, >, and = and justify the conclusions using a visual fraction model.