Unpacked South Dakota State Mathematics Standards

Purpose: *In order for students to have the best chance of success, standards, assessment, curriculum resources, and instruction must be aligned in focus, coherence, and rigor. Unpacked standards documents are intended to help align instruction to the focus, coherence, and rigor of the South Dakota State Mathematics Standards. The standards have been organized in clusters as they are not so much built from topics, but rather woven out of progressions. Not all content in a given grade is emphasized equally in the mathematics standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting standards will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.*

Domain: Number System		**Grade Level: 6**
6.NS.A Cluster: Apply and extend previous understandings of multiplication and division to divide fractions by fractions. Students will continue their previous understanding of the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to explain why the procedures for dividing fractions make sense. They use visual models and equations to divide whole numbers by fractions and fractions by fractions to solve word problems.		
****This is a MAJOR cluster.** *Students should spend the large majority of their time (65-85%) on the major work of the grade. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.* **6.NS.1** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?		
Aspects of Rigor for Students: (Conceptual, Procedural, and/or Application)		
Understand the meaning behind dividing fractions by fractions (e.g. by using visual models and equations). **(6.NS.1)** Represent fraction by fraction division problems using visual models. **(6.NS.1)** Use the relationship between multiplication and division to explain problems (e.g. use fact families) **(6.NS.1)**	Compute division of fraction by fraction problems. **(6.NS.1)**	Solve real-world and mathematical problems involving division of fractions by fractions. **(6.NS.1)**
Enacting the Mathematical Practices - Evidence of Students Engaging in the Practices		
1. **Make sense of problems and persevere in solving them.** ● Interpret and make sense of a problem involving division of fractions. 2. **Reason abstractly and quantitatively.** ● Use the meaning of fractions, the meaning of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. ● Understand the meaning of a division problem.		

3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
    - Use manipulatives to model everyday problems with fractions.
    - Model estimating the sum, difference, product or quotient from the problems before performing the operation.
5. **Use appropriate tools strategically.**
    - Use fraction models, including manipulatives and visual diagrams to interpret, represent, and solve real world problems of division and fractions.
6. **Attend to precision.**
    - Students communicate precisely with others and use clear mathematical language when discussing the understanding and procedure for dividing fractions.
7. **Look for and make use of structure.**
    - Apply division algorithms to divide fractions by fractions.
8. **Look for and express regularity in repeated reasoning.**
    - Consider the reasonableness of an estimated quotient.

Vertical and Horizontal Coherence and Learning Progressions

Previous Learning Connections	***Current Learning Connections***	***Future Learning Connections***
In Grade 3, learners understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8 **AND** interpret whole-number quotients of whole numbers (e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.) In Grade 5, learners apply what they learned previously to divide unit fractions by whole numbers and whole numbers by unit fractions.	The conceptual understanding of the distributive property comes in **6.EE.A.**	In Grade 7 learners apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. In Grade 7, learners solve real-world and mathematical problems involving the four operations with rational numbers. In high school, learners continue to use their understanding of division of fraction knowledge when solving more complex algebraic equations.

Vocabulary *(Key Terms Used by Teachers and Students in this Cluster):*

- Numerator
- Denominator
- Reciprocal
- Quotient
- Multiplicative Inverse
- Division
- Dividend
- Divisor
- Quotient
- Algorithm
- Estimate
- Multi-digit Decimals
- Distributive Property
- Greatest Common Factor
- Least Common Multiple
- Multiple
- Prime Factorization
- Factor Trees

Relevance, Explanations, and Examples:

Time needs to be spent conceptually learning the meaning of fraction by fraction division using visual models. This will lead into the algorithm using the reciprocal or by using common denominators (see example below). The standard using factors and multiples **(6.NS.4)** is a supporting standard and can be woven into **6.NS.1**, especially when using the common denominators method of dividing.

Multiplicative Inverse is a synonym for reciprocal.

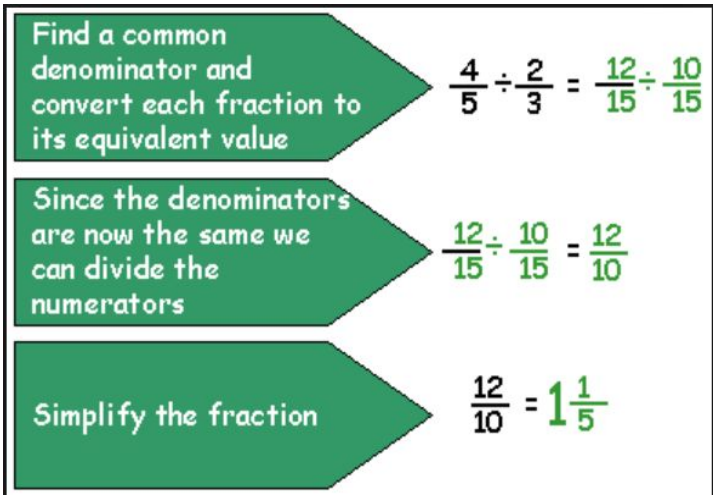
Visual model for $\frac{2}{3} \div \frac{3}{4}$ and $\frac{3}{4} \times ? = \frac{2}{3}$

How many cups of yogurt?

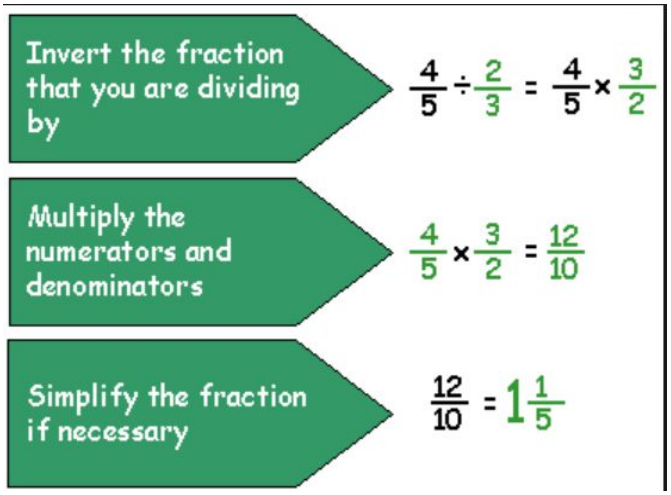
$\frac{2}{3}$ of a cup of yogurt

The shaded area is $\frac{3}{4}$ of the entire strip. So it is 3 parts of a division of the strip into 4 equal parts. Another way of seeing this is that the strip is 4 parts of a division of the shaded area into 3 equal parts. That is, the strip is $\frac{4}{3}$ times the shaded part. So $? = \frac{4}{3} \times \frac{2}{3} = \frac{8}{9}$.

Common Denominator Division Method

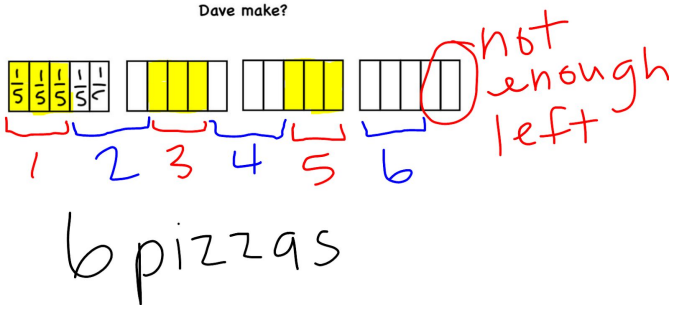


Standard Algorithm (Multiplicative Inverse) Method



Another way of thinking about dividing fractions is: how many times does a/b fit in to c/d? In the example below, they would set it up as 4 ÷ ⅘ and ask the question: how many times does ⅘ fit into 4?

Dave is making pizza. He has four blocks of cheese. If each pizza needs 3/5 of a block of cheese, how many pizzas can Dave make?



Achievement Level Descriptors	
Cluster: **Apply and extend previous understandings of multiplication and division to divide fractions by fractions.**	
Concepts and Procedures	***Level 1:*** Students should be able to apply and extend previous understandings of multiplication and division to multiply a fraction by a fraction, divide a fraction by a whole number, and be able to connect to a visual model. They should

	understand the effect that a fraction greater than or less than 1 has on a whole number when multiplied and use or create visual models when multiplying a whole number by a fraction between 0 and 1.
	Level 2: Students should be able to apply and extend previous understandings of multiplication and division to divide a whole number by a fraction between 0 and 1, divide a mixed number by a whole number, and be able to connect to a visual model.
	Level 3: Students should be able to apply and extend previous understandings of multiplication and division to divide a fraction by a fraction and be able to connect to a visual model
	Level 4: Students should be able to use visual models in settings where smaller fractions are divided by larger fractions. They should also understand and apply the fact that a fraction multiplied or divided by 1 in the form of a/a is equivalent to the original fraction.