Unpacked South Dakota State Mathematics Standards

Purpose: *In order for students to have the best chance of success, standards, assessment, curriculum resources, and instruction must be aligned in focus, coherence, and rigor. Unpacked standards documents are intended to help align instruction to the focus, coherence, and rigor of the South Dakota State Mathematics Standards. The standards have been organized in clusters as they are not so much built from topics, but rather woven out of progressions. Not all content in a given grade is emphasized equally in the mathematics standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting standards will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.*

Domain: Ratios and Proportional Relationships	**Grade Level: 6**

6.RP.A Cluster: Understand ratio concepts and use ratio reasoning to solve problems.

Students are introduced to ratios, a relationship or comparison of two quantities or measures. They will represent ratios in various forms (a:b, a to b, a/b) and compare types of ratios. They will use reasoning about multiplication and division to solve ratio and rate problems about quantities. Students will learn how and where ratios and rates are used in the real world.

****This is a MAJOR cluster.** *Students should spend the large majority of their time (65-85%) on the major work of the grade. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.*

6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received three votes."

6.RP.2 Understand the concept of a unit rate a/b associated with a ratio a:b with b not equal to 0, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?
c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Aspects of Rigor for Student Learning: (Conceptual, Procedural, and/or Application)

Conceptual Understanding	***Procedural Fluency***	***Application***
Understand that a ratio is a comparison of two quantities. **(6.RP.1)** Describe what a ratio illustrates using ratio language. **(6.RP.1)**	Write a ratio relationship in the forms a:b, a to b, a/b. **(6.RP.1)**	

Translate a ratio relationship into words. **(6.RP.1)** Understand the differences between part:part and part:whole relationships. **(6.RP.1)**		
Understand that a unit rate compares two quantities with different units of measure. **(6.RP.2)** Recognize that using the terms "per," "each" or "for every" is an indicator of unit rate. **(6.RP.2)** Understand that unit rate ratios can be written as fractions where the denominator cannot be zero. **(6.RP.2)** Use reasoning to solve for unit rate. **(6.RP.2)**		
	Solve ratio and rate reasoning problems with tables, equivalent ratios, tape diagrams, double line diagrams, or equations. **(6.RP.3a)** Use tables to compare ratios. **(6.RP.3a)** Create and interpret a table of equivalent ratios. **(6.RP.3a)** Plot pairs of values from a table to a coordinate plane. **(6.RP.3a)**	Use ratio language to describe real-world experiences and use understanding for decision making. **(6.RP.3a)**
	Solve and simplify unit rate problems. **(6.RP.3b)**	Apply unit rate understanding to find unit pricing, constant speed and other real-world and mathematical problems. **(6.RP.3b)** Reason to determine the better buy. **(6.RP.3b)**
Understand that a percent of a quantity is a rate per 100. **(6.RP.3c)**	Find the percent of a quantity. (e.g., 30% of a quantity means 30/100 times the quantity). **(6.RP.3c)** Find the whole when given a part and the percent. (e.g., 20 is 50% of what number). **(6.RP.3c)**	Apply percent understanding to solve real-world problems involving finding the whole when given a part and the percent. **(6.RP.3c)**
Use reasoning to convert between measurement units. **(6.RP.3d)**	Manipulate and transform between units by using multiplication and division. **(6.RP.3d)**	Apply ratio reasoning to convert between measurement units with real-world or mathematical problems. **(6.RP.3d)**

Enacting the Mathematical Practices - Evidence of Students Engaging in the Practices

1. **Make sense of problems and persevere in solving them.**
   - Interpret and solve ratio problems.
2. **Reason abstractly and quantitatively.**
   - Solve problems by analyzing and comparing ratios and unit rates in tables, equations and graphs.
3. **Construct viable arguments and critique the reasoning of others.**
   - Allow students to talk with each other and teacher to make sense of what they are learning.
4. **Model with mathematics.**
   - Model real-life situations with mathematics and model ratio problem situations symbolically.
5. **Use appropriate tools strategically.**
   - Use tape diagrams, coordinate planes and tables to help students find equivalent ratios, missing values and compare ratios and rates.
6. **Attend to precision.**
   - Use clear mathematical language when describing a ratio relationship between quantities.
7. **Look for and make use of structure.**
   - Recognizing ratios, rates, and proportional relationships involves looking for structure.
8. **Look for and express regularity in repeated reasoning.**
   - Provide cyclical, distributed practice over time to continually practice unit rate problems.

Vertical and Horizontal Coherence and Learning Progressions

Previous Learning Connections	***Current Learning Connections***	***Future Learning Connections***
In Grade 4, learners have recorded measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36)... In Grade 4, learners were taught to multiply or divide to solve word problems involving multiplicative comparison. In Grade 5, learners had to interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b) AND interpret multiplication as scaling, or resizing. In Grade 5, learners represented real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	Learners will need to understand the relationships between numbers as they use variables to represent two quantities in a real-world problem that change in relationship to one another. **(6.EE.C.9)**	In Grade 7, learners will need to compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. In Grade 7, learners will recognize and represent proportional relationships between quantities. In Grade 7, learners will use proportional relationships to solve multistep ratio and percent problems. In high school, learners will apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). In high school, learners will use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

Vocabulary *(Key Terms Used by Teachers and Students in this Cluster):*

- Ratio
- Compare
- Simplify
- Rates
- Unit Rates
- Equivalent Ratios
- Percents
- Ratio Tables
- Tape Diagrams

Relevance, Explanations, and Examples:

Part : Whole - 7 of our 15 students take music class

Part : Part - the ratio of white socks to red socks in the drawer is 5:8 (a part of one whole compared to another part of that same whole)

Representing ratios with double number line diagrams

On double number line diagrams, if A and B are in the same ratio, then A and B are located at the same distance from 0 on their respective lines. Multiplying A and B by a positive number p results in a pair of numbers whose distance from 0 is p times as far. So, for example, 3 times the pair 2 and 5 results in the pair 6 and 15 which is located at 3 times the distance from 0.

Representing a problem with a tape diagram

Slimy Gloopy mixture is made by mixing glue and liquid laundry starch in a ratio of 3 to 2. How much glue and how much starch is needed to make 85 cups of Slimy Gloopy mixture?

5 parts $\longrightarrow$ 85 cups
1 part $\longrightarrow$ $85 \div 5 = 17$ cups
3 parts $\longrightarrow$ $3 \cdot 17 = 51$ cups
2 parts $\longrightarrow$ $2 \cdot 17 = 34$ cups

51 cups glue and 34 cups starch are needed.

Tape diagrams can be useful aids for solving problems.

A progression of strategies for solving a proportion

If 2 pounds of beans cost \$5, how much will 15 pounds of beans cost?

Method 1

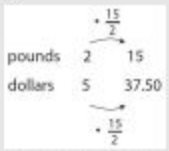
pounds	2	4	6	8	10	12	14	1	15
dollars	5	10	15	20	25	30	35	2.50	37.50

"I found 14 pounds costs \$35 and then 1 more pound is another \$2.50, so that makes \$37.50 in all."

Method 2

"I found 1 pound first because if I know how much it costs for each pound then I can find any number of pounds by multiplying."

Method 3



The previous method, done in one step.

With this perspective, the second column is seen as the first column times a number. To solve the proportion one first finds this number.

Solving a percent problem

If 75% of the budget is \$1200, what is the full budget?

"I said 75% is 3 parts and is \$1200
25% is 1 part and is $\$1200 \div 3 = \$400$
100% is 4 parts and is $4 \cdot \$400 = \$1600$"

portion	75	3	1200
whole	100	4	1600

75% is $\frac{1200}{B}$

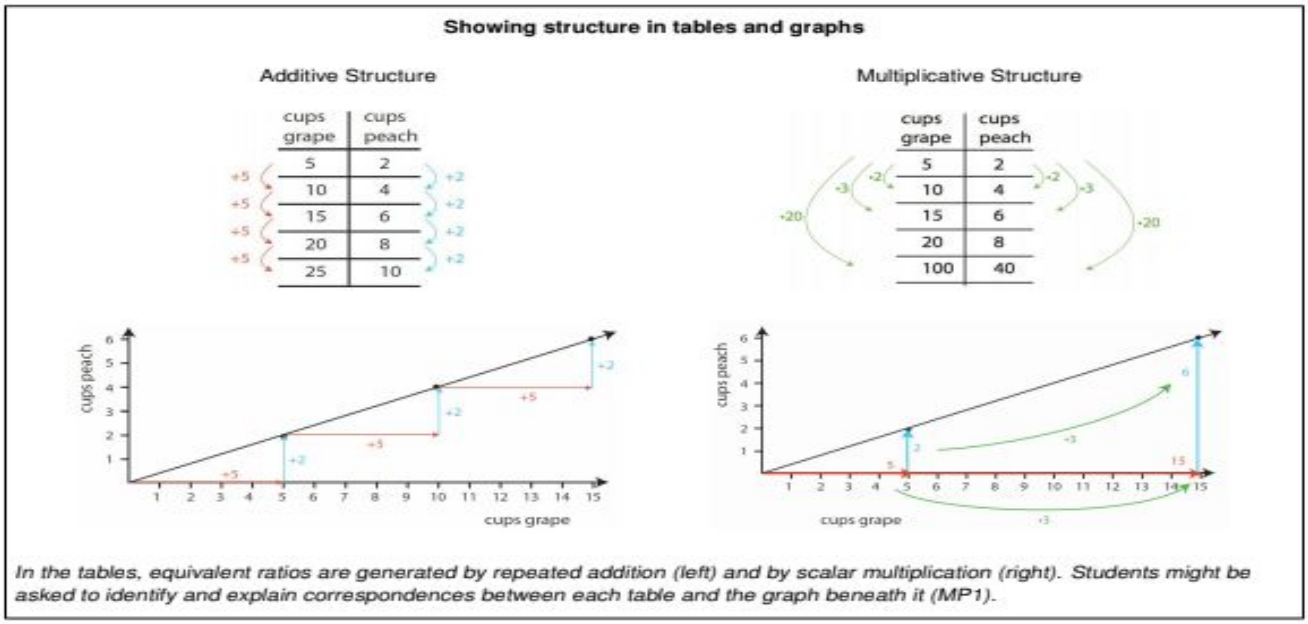
$\frac{75}{100} = \frac{1200}{B}$

$B = 1600$

75% of B is 1200

$\frac{75}{100} \cdot B = 1200$

In reasoning about and solving percent problems, students can use a variety of strategies. Representations such as this, which is a blend between a tape diagram and a double number line diagram, can support sense-making and reasoning about percent.



It is not specifically written in to the standards, but this would be a great place to introduce converting between fractions, decimals, and percents. Learners will really dig in to this concept in 7th grade. Have students represent 30% by filling in 30 units on a hundreds grid. Have students write as a fraction (30/100), a decimal (.30) and a percent (30%). **(6.RP.3c)**

Measurement conversion provides other opportunities for students to use relationships given by unit rates. (Example: 12 inches per 1 foot - as a rate can help connect concepts and methods developed for other contexts with measurement conversion.) **(6.RP.3d)**

Achievement Level Descriptors	
Cluster: **Understand ratio concepts and use ratio reasoning to solve problems.**	
Concepts and Procedures	**Level 1:** Students should be able to describe a ratio relationship between two whole number quantities, find missing values in tables that display a proportional relationship, and plot the pairs of values from a table on the Coordinate plane. They should be able to find a percent as a rate per hundred and convert measurement units.
	Level 2: Students should be able to understand the concept of unit rate in straight forward, well-posed problems and solve straight forward, well-posed, one-step problems requiring ratio reasoning Threshold: Find unit rates given two whole number quantities where one evenly divides the other.
	Level 3: Students should be able to use ratio reasoning to solve and understand the concept of unit rates in unfamiliar or multi-step problems, including instances of unit pricing and constant speed, and solve percent problems by finding the whole, given a part and the percent. They should be able to describe a ratio relationship between any two number quantities (denominators less than or equal to 12). Threshold: The student who just enters Level 3 should be able to: ● Solve unit rate problems. ● Solve percent problems by finding the whole, given a part and the

	percent. ● Describe a ratio relationship between any two number quantities and understand the concept of unit rate in problems (denominators less than or equal to 12).
	Level 4: Students should be able to solve unfamiliar or multi-step problems by finding the whole, given a part and the percent; explain ratio relationships between any two number quantities; and identify relationships between models or representations. Threshold: Solve unfamiliar or multi-step problems by finding the whole, given a part and the percent. ● Understand and explain ratio relationships between any two number quantities. ● Understand and apply the fact that a fraction multiplied or divided by 1 in the form of a/a is equivalent to the original fraction.