Unpacked South Dakota State Mathematics Standards

Purpose: In order for students to have the best chance of success, standards, assessment, curriculum resources, and instruction must be aligned in focus, coherence, and rigor. Unpacked standards documents are intended to help align instruction to the focus, coherence, and rigor of the South Dakota State Mathematics Standards. The standards have been organized in clusters as they are not so much built from topics, but rather woven out of progressions. Not all content in a given grade is emphasized equally in the mathematics standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting standards will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.

Domain: Functions	Grade Level: Algebra 2

A2.F.IF.B Cluster: Analyze Functions Using Different Representations

Learners make sense of problems by comparing different representations of functions, discovering features of families of functions, and finding key features such as zeros or extrema by graphing, symbol manipulation, or tables.

**This is a MAJOR cluster. *Students should spend the large majority of their time (65-85%) on the major work of the grade. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.*

A2.F. IF.B.7.(ii) Graph parent functions and their transformations expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. (Uses Modeling)

    b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

    c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

    d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available and showing end behavior.

    e. Graph logarithmic functions, showing intercepts and end behavior.

    f. Graph trigonometric functions (sine and cosine), showing period, midline, and amplitude.

A2.F. IF.B.9.(ii) Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Aspects of Rigor for Students: (Conceptual, Procedural, and/or Application)

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f. Graph trigonometric functions (sine and cosine), showing period, midline, and amplitude.

Conceptual Understanding	*Procedural Fluency*	*Application*
Learners understand how each symbolic change made to the equation of the parent function results in a specific transformation of the graph of the parent function. Learners understand in functions there is an underlying structure that determines the transformation of any function, regardless of its type. For example, changing the leading coefficient of a function will always result in a vertical stretch or vertical shrink of the function's graph. See example below in **Relevance, Explanations, and Examples**	Learners are able to identify and graph parent functions and their transformations (i.e. vertical translation, horizontal translation, vertical stretch/shrink, reflect over x-axis, etc.).	Given a function, learners will be able to model that function in graphic form.

**A2.F. IF.B.9.(ii) Compare properties of two functions each represented in a different way (algebraically, graphically,
numerically in tables, or by verbal descriptions).**

Conceptual Understanding	*Procedural Fluency*	*Application*
Learners understand and explain how key features of different functions compare. The key is that the functions be represented in different ways (i.e. one function may be presented graphically while the other function may be represented in a table).		

Enacting the Mathematical Practices - Evidence of Students Engaging in the Practices

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
   - Students will be able to compare two functions (represented in different ways) and their key features.
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
   - Students will be able to understand the general shape of all parent functions (given in any form - equation, table, verbal description) and the underlying structure of transformations of all functions.
8. **Look for and express regularity in repeated reasoning.**
   - Students will be able to apply transformations to basic parent functions to graph other functions in the family.

Vertical and Horizontal Coherence and Learning Progressions

| *Previous Learning Connections*	*Current Learning Connections*	*Future Learning Connections*

In Algebra 1, students have explored transformations with linear, quadratic and exponential functions. Students have also compared key features of these functions in different representations (symbolically, in a table, and graphically). *In Geometry, students discuss transformations on a coordinate plane.*	*This cluster transcends through the entire course, since it is discussed with every new function presented. This cluster also connects to the transformations of functions, and the concept of building new functions from existing functions.*	*Subsequent math course build upon these key features and transformations of relations (both functions and non-functions).*

### *Vocabulary (key terms and definitions)*

- Horizontal translation
- Parent function
- Reflection over the x-axis
- Transformations

- Vertical Translation
- Vertical shrink
- Vertical stretch

Relevance, Explanations, and Examples:

Example:
State the transformations from the parent function and graph the transformed function. $f(x) = 2\sqrt{x+4} - 1$
The function $y = \sqrt{x}$ is vertically stretched by a factor of 2, horizontally translated to the left 4 units, and vertically translated down 1 unit. (In the graph below, the parent function is graphed in black and the transformed graph is in red)
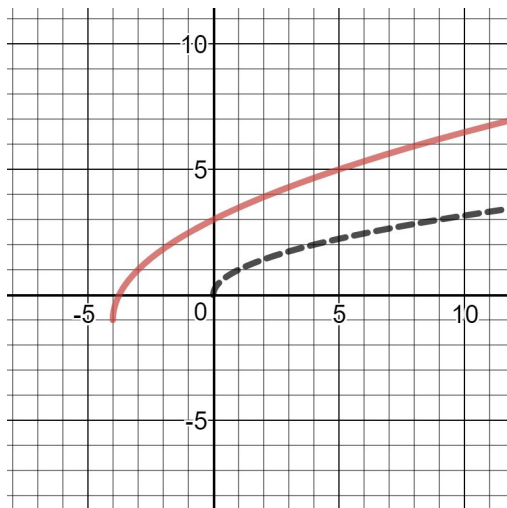
Similarly- if you look at the function $f(x) = 2(x+4)^2 - 1$, the function $f(x) = x^2$ is vertically stretched by a factor of 2, horizontally translated to the left 4 units, and vertically translated down 1 unit. (In the graph below, the parent function is graphed in black and the transformed graph is in red)
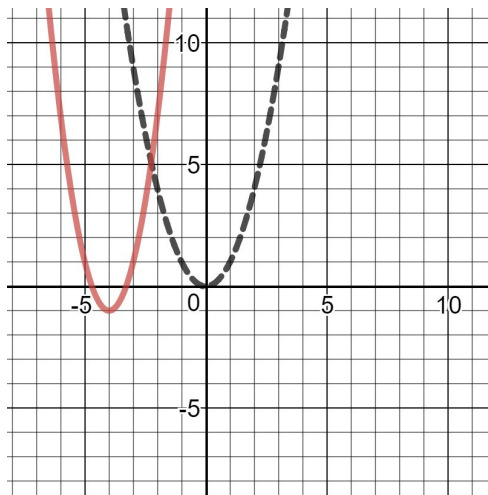
Note that the 2, the 4, and the -1 result in the same types of transformations in both the square root function and the quadratic function.

Achievement Level Descriptors

Cluster: Analyze Functions Using Different Representations

Concepts and Procedures	**Level 1:** Students should be able to graph a linear function by hand or by using technology. They should be able to compare properties of two linear functions represented in different ways. They should be able to identify equivalent forms of linear functions.
	Level 2: Students should be able to graph linear and quadratic functions by hand; graph square root, cube root, piecewise-defined, polynomial, exponential, and logarithmic functions by hand or by using technology; compare properties of two quadratic or two other functions of the same type, i.e., linear to linear, represented in different ways; and understand equivalent forms of linear and quadratic functions. They should be able to compare properties of two trigonometric functions represented in the same way.
	Level 3: Students should be able to analyze and compare properties of two functions of different types represented in different ways and understand equivalent forms of functions. They should be able to graph trigonometric functions by hand and by using technology.
	Level 4: Students should be able to graph a variety of functions, including linear, quadratic, square root, cube root, piecewise-defined, polynomial, exponential, logarithmic, and trigonometric, by hand and by using technology. They should be able to analyze and explain relationships between various types of functions and the behaviors of the functions and be able to determine which equivalent form is most appropriate for a given task.